CERTIFIED ACCOUNTING TECHNICIAN

LEVEL 1 EXAMINATION

L1.4: BUSINESS MATHEMATICS

WEDNESDAY: 5 JUNE 2019

INSTRUCTIONS:

1. **Time Allowed: 3 hours 15 minutes** (15 minutes reading and 3 hours writing).
2. This examination has **seven** questions and only **five** questions are to be attempted.
3. Marks allocated to each question are shown at the end of the question.
4. Show all your workings, where applicable.

QUESTION ONE

(a) Distinguish between an equation and an inequality as used in basic mathematics. **(4 Marks)**

(b) Solar Power (R) Ltd wishes to sell some of its shares to the public. The company is contemplating what the initial public offering would be. Mr. Bikindi has developed the inequality below to help the company determine the value of a share, p in Francs of each share $\frac{3p-1}{p+2} \geq \frac{5}{4}$.

REQUIRED:

Determine the minimum value of p at initial public offering by solving the inequality **(5 Marks)**

(c) A lake is polluted by waste from a factory located on its shores. Ecologists have warned that when the level of pollutant is x per a thousand cubic metres of water, there will be only F fish of a certain species in the lake, where $F = \frac{32{,}000}{3+\sqrt{x}}$.

REQUIRED:

Determine the level of pollutant x when there are 4,000 fish.

(5 Marks)

(d) Bugingo Furniture Mart Ltd (BFML) makes chairs and tables for sale to retail outlets. A complete chair requires two hours of assembling and three hours of finishing while a table requires three hours of assembling and five hours of finishing. In a week BFML has 60 hours of assembling and 95 hours for finishing available.

REQUIRED:

(i) Form simultaneous equations to illustrate the number of tables and chairs that can be made within the available labour hours.

(3 Marks)

(ii) Solve the equations in (d) (i) above by elimination method to obtain the number of chairs and tables made within the week.

(3 Marks)

(Total 20 Marks)

QUESTION TWO

(a) Ruzibiza Baptiste is an online mobile car vendor dealing in Toyota models of Spacio and Fielder. In order to boost his sales, he designed an online lottery giving a 10% discount on the Fielder model. A client can only know the model after placing the online bid using the lottery ticket and the ticket is valid for a maximum of three bids.

Given that the probability of obtaining a Spacio model is twice that of obtaining a Fielder model;

REQUIRED:

(i) Represent the given information on a tree diagram. **(3 Marks)**

(ii) Compute the probability of obtaining a Fielder model if the client is willing to utilize all the three bids?

(3 Marks)

(b) 70 college students undertook their final internship at the headquarters of Rwanda Revenue Authority (RRA). At the end of the internship period, an analysis was done on the trainee participation in the three departments of audit, tax and corporate services and the following results were obtained:

41 trained under the audit department,
44 trained under the tax department,
44 trained under the corporate services department,
14 trained under all the three departments,
24 trained under both the audit and tax departments,
26 trained under both the audit and corporate services departments,
2 trained under corporate services department only.

REQUIRED:

(i) Represent the given information on a Venn diagram **(3 Marks)**

(ii) Obtain the number of trainees that trained under only one department.

(2 Marks)

(iii) Determine the number of trainees that did not participate in any of the departmental trainings availed.

(2 Marks)

(c) Ganza Olivier & Co. Ltd is a plastic bottle manufacturing company. Each bottle produced is tested to ensure that it meets international standards. The production unit has established that 40% of the bottles manufactured fail the test. A sample of 10 bottles is availed to the Standards Department for testing.

REQUIRED:

Determine the:

(i) Probability that less than 3 bottles failed the test. **(5 Marks)**

(ii) Expected number of bottles that passed the test. **(2 Marks)**

(Total 20 Marks)

QUESTION THREE

(a) Define the term 'primary data' and identify one of its sources. **(2 Marks)**

(b) Charity Akimana is a businesswoman resident at Byumba in North province. She is planning to establish a sand supply centre so as to meet the demand of the real estate developers in Byumba. A feasibility study she conducted revealed that the weekly supply of sand by various agents, in tons, is as indicated in the following table:

Weekly supply per truck	Cumulative number of trucks
200-209	2
210-219	6
220-229	13
230-239	25
240-249	35
250-259	43
260-269	49
270-279	50

REQUIRED:

Determine the standard deviation of the weekly supply of sand. **(6 Marks)**

(c) Ntampaka & Associates Fruit Growers (NAFG) package and export fruits in boxes whose weights are normally distributed, with mean 30.2kg and standard deviation 1.04kg. International market regulations demand that a box is of standard weight if it is between 28kg and 32kg. NAFG are to meet a weekly quota of 1,250 boxes for export.

REQUIRED:

Determine the:

(i) probability that a fruits box meets international market regulations. **(4 Marks)**

(ii) number of boxes that do not meet the standard weight requirement. **(3 Marks)**

(d) Toto Divine Enterprises Ltd (TDEL) deal in fabric import business with its headquarters at Kigali. They have regional offices with employee statistics shown in the table below:

Category	Number of employees
Management	32
Technical	152
Supervisor	48
Others	134

REQUIRED:

(i) Represent TDEL's employee statistics on a Pareto chart. **(3 Marks)**
(ii) If the ideal ratio of top management (management and supervisors) to the rest of the employees is 1:4, advise the owners of TDEL on their employment record.
(2 Marks)
(Total 20 Marks)

QUESTION FOUR

(a) Explain **three** limitations of index numbers. **(3 Marks)**
(b) Nelson Gatera manufactures juice called Agasha. His juice sales (in '000' litres) recorded for the last ten months are as follows.

Month	Sales (in '000' litres)
January	92
February	80
March	76
April	58.5
May	44
June	54
July	51.5
August	62.5
September	31
October	45

REQUIRED:

(i) Calculate the trend values of sales using 4-centered moving averages.
(5 Marks)
(ii) Plot a scatter of sales against time. **(2 Marks)**
(iii) From the scatter, draw the line of best fit for the sales and use it to predict the sales in November.
(2 Marks)
(iv) Find the equation of the line of best fit. **(3 Marks)**

(c) The Rwanda National Forest Authority (RNFA) is mandated to regulate, protect and manage forest resources in the country. In its regulatory role, the following data about the production of timber (in tons) was collected from Western Province from 2000 to 2018.

Year	2000	2002	2004	2006	2008	2010	2012	2014	2016	2018
Production	37.0	39.4	40.5	40.6	35.4	31.7	35.0	34.6	37.1	36.3

REQUIRED:

Using 2012 as base year, construct a fixed base index for the timber produced from 2010 to 2018.

(5 Marks)
(Total 20 Marks)

QUESTION FIVE

(a) Explain the phrase 'additive model' as used in time series. **(1 mark)**

(b) Dr. Ingabire is undertaking a survey on the relationship between the number of years that a nurse is employed (x) and the number of nurses who voluntarily resign (y). The following table shows the data that she collected from Nyanza Main Hospital in South province.

x	2	3	5	6	7	8
y	8	7	7	4	1	2

REQUIRED:

(i) Calculate the linear regression line y= a +bx. **(8 marks)**
(ii) Comment on your result in (b) (i) above. **(1 mark)**

(c) Hitimana Aquatics Ltd (HAL) in Kigali rears and sells fish. The following table shows quarterly sales (in Frw 'million') of fish for the period 2015 to 2018.

Quarter	2015	2016	2017	2018
1	2.6	3.6	4.9	7.2
2	2.9	4.1	5.8	7.5
3	3.5	4.8	6.4	8.2
4	5.7	6.9	8.8	10.7

REQUIRED:

Use the additive model to compute the seasonal variations. **(10 marks)**
(Total 20 Marks)

QUESTION SIX

(a) Distinguish between nominal and effective rate of interest. **(2 marks)**

(b) Justine Nzayisenga and Bosco Imana are fresh university graduates. They have each saved Frw 112,500 and are planning to start a joint investment after 4 years. To improve their capital, they are to invest their savings for 4years.

Nzayisenga has decided to invest with an investment club which offers a compound interest of 10.5% per annum compounded bi-annually for 3½ years. Imana on the other hand has negotiated with a commercial bank for a compound interest of 8% and will also save a fixed amount of Frw 15,000 at the end of each year for 4 years.

REQUIRED:

Determine the total capital generated by the two partners at the end of the 4 years.

(7 Marks)

(c) Pius Ankunda has invested in transport business. In a bid to grow his vehicle fleet, he took up a car loan of Frw 10 million at compound interest of 12% per annum for a period of 1½ years, compounded quarterly.

REQUIRED:

(i) Calculate the quarterly payments to be made to amortize the car loan. **(4 marks)**

(ii) Determine the interest accrued. **(2 marks)**

(iii) Prepare the amortization schedule for the car loan. **(5 marks)**

(Total 20 Marks)

QUESTION SEVEN

(a) Explain any **four** disadvantages of spreadsheets. **(4 marks)**

(b) The maturity value of a deposit P in the bank at a rate of interest per annum R, after time T is connected by the formula $M = P(1+RT)$, which appears in one of the cells in a spread sheet.

REQUIRED:

(i) Express T in terms of M, P and R. **(3 marks)**

(ii) If at maturity the value in the cell is Frw 2,000; determine T (in months) if P = Frw 500 and $R = 0.1$

(2 marks)

(c)

(i) Explain the term 'discounted cash flow' as applied in business. **(2 marks)**

Gasigwa is planning to invest in an agricultural farm on condition that its net present value (NPV) is above Frw 5,000,000. Statistics show the actual cash flows for the 4 year plan as illustrated below:

Year	Actual cash flow
	Frw '000'
1	2,000
2	4,000
3	4,000
4	2,000

REQUIRED:

'

(ii) Determine the discounted cash flow for each year using a discount rate of 10%. **(4 marks)**

(iii) Determine the NPV if the business requires an initial investment of Frw 3,000,000 and advise Gasigwa on his investment plan. **(5 marks)**

(Total 20 Marks)

FORMULAE

1.	Combination ${}^nC_r = \frac{n!}{(n-r)!r!}$				
2.	Permutations ${}^n p_r = \frac{n!}{(n-r)!}$				
3.	Mean of the binomial distribution= np				
4.	Standard deviation = $\sqrt{npq}$				
5.	Variance of the binomial distribution $= np(1-p)$				
6.	Standard error of population proportion $S_{ps} = \sqrt{\frac{pq}{n}}$				
7.	Spearman's rank correlation coefficient $r = 1 - \frac{6\sum d^2}{n(n^2-1)}$				
8.	Product moment coefficient of correlation = $\frac{n\sum xy - \sum x \sum y}{\sqrt{(n\sum x^2 - (\sum x)^2) \times (n\sum y^2 - (\sum y)^2)}}$				
9.	Cost slope	=	crash cost – normal cost / normal time – crash time		
10.	Harmonic mean (ungrouped data) $hm = \frac{n}{\sum \frac{1}{x}}$				
11.	Sample mean	$\bar{x} = \frac{\sum x}{n}$			
12.	Harmonic mean (grouped data) $hm = \frac{n}{\sum \frac{f}{x}}$				
13.	Quartile coefficient of dispersion $= \frac{Q_3 - Q_1}{Q_3 + Q_1}$				
14.	Mean $\bar{x} = A + \frac{\sum fd}{\sum f}$		or	Mean $\bar{x} = \frac{\sum fx}{\sum f}$	
15.	Median $= Lb + \left(\frac{\frac{N}{2} - Cfb}{fm} \right) C$				
16.	Mode $= lm + \left(\frac{d_1}{d_1 + d_2} \right) C$				
17.	Variance $Var(x) = \frac{\sum fx^2}{\sum f} - \left(\frac{\sum fx}{\sum f} \right)^2$				
18.	Standard deviation	$\delta = \sqrt{\frac{\sum fx^2}{\sum f} - \bar{x}^2}$	$= \sqrt{\frac{\sum f(x-\bar{x})^2}{\sum f}}$	$\sqrt{\left(\frac{\sum fd^2}{\sum f} \right) - \left(\frac{fd}{\sum f} \right)^2}$	

19.	Sample standard deviation	$s=\sqrt{\frac{\sum(x-\bar{x})^2}{n-1}}$		
20.	Least squares regression equation of y on x is given by; $y=a+bx$			
21.	Where; $b=\frac{n\sum xy-\sum x\sum y}{n\sum x^2-\left(\sum x\right)^2}$	and	$a=\frac{\sum y}{n}-\frac{b\sum x}{n}$	
22.	Least squares regression equation of x on y is given by; x = c +dy			
	Where $c=\frac{\sum x}{n}-\frac{d\sum y}{n}$	and	$d=\frac{n\sum xy-\sum x\sum y}{n\sum y^2-(\sum y)^2}$	
23.	Standardizing normal.		$z=\frac{\bar{x}-\mu}{\sigma}$	
24.	Confidence interval for sample mean $=\bar{x}\pm t_{\alpha/2}\frac{s}{\sqrt{n}}$			
25.	$\chi^2=\sum\frac{(O-E)^2}{E}$			
26.	Confidence interval of proportion $=p\pm z_{\alpha/2}\sqrt{\frac{pq}{n}}$			
27.	Pearson coefficient of skewness	$Sk=\frac{(\bar{x}-\text{mode})}{s_d}$	or	$Sk=\frac{3\left(\bar{x}-median\right)}{s_d}$
28.	Expectation $=\sum xP(X=x)$			
29.	Laspeyres' price index $=\frac{\sum(p_1\times q_0)}{\sum(q_0\times p_0)}\times 100$			
30.	Weighted aggregate price index $=\frac{\sum wv_n}{\sum wv_0}\times 100$			
31.	Additive law of probability; $P(A\cup B)=P(A)+P(B)-P(A\cap B)$			
32.	Conditional probability $P\left(A/B\right)=\frac{P(A\cap B)}{P(B)}$			
33.	Independence of A, B $P\left(A/B\right)=P(A)or P(A\cap B)=P(A)\times P(B)$			
34.	Continuous compounding $A=P(1+r)^n+\frac{b(1+r)^n-b}{r}$			
35.	Quotient rule of differentiation $f=\frac{vu^1-uv^1}{v^2}$; where $f=\frac{u}{v}$			
36.	$Paasche'sModel=\frac{\sum(p_1\times q_1)}{\sum(q_1\times p_0)}\times 100$			

37.	$PoissonModelP(X = x) = e^{-\lambda}\frac{\lambda^x}{x!}$
38.	Price relative index= $\frac{p_n}{p_0}x100$
39.	**Discounted cash flow**, $DCF = \frac{CF}{(1+r)^n}$
40.	**Net Present Value**, $NPV = \sum DCF - IV$

CUMULATIVE NORMAL DISTRIBUTION $P(z)$

Z	0	1	2	3	4	5	6	7	8	9	ADD 1	2	3	4	5	6	7	8	9
0.0	0.0000	0040	0080	0120	0160	0199	0239	0279	0319	0359	4	8	12	16	20	24	28	32	36
0.1	0.0398	0438	0478	0517	0557	0596	0636	0675	0714	0753	4	8	12	16	20	24	28	32	36
0.2	0.0793	0832	0871	0910	0948	0987	1026	1064	1103	1141	4	8	12	15	19	22	27	31	35
0.3	0.1179	1217	1255	1293	1331	1368	1406	1443	1480	1517	4	8	11	15	19	22	26	30	34
0.4	0.1554	1591	1628	1664	1700	1736	1772	1808	1844	1879	4	7	11	14	18	22	25	29	32
0.5	0.1915	1950	1985	2019	2054	2088	2123	2157	2190	2224	3	7	10	14	17	21	24	27	31
0.6	0.2257	2291	2324	2357	2389	2422	2454	2486	2517	2549	3	6	10	13	16	19	23	26	29
0.7	0.2580	2611	2642	2673							3	6	9	12	15	19	22	25	28
					2704	2734	2764	2794	2823	2852	3	6	9	12	15	18	21	24	27
0.8	0.2881	2910	2939	2967	2995	3023					3	6	8	11	14	17	20	22	25
							3051	3078	3106	3133	3	5	8	11	13	16	19	22	24
0.9	0.3159	3186	3212	3238	3264	3289					3	5	8	10	13	16	18	21	23
							3315	3340	3365	3389	2	5	7	10	12	15	17	20	22
1.0	0.3413	3438	3461	3485	3508						2	5	7	10	12	14	17	19	22
						3531	3554	3577	3599	3621	2	4	7	9	11	13	15	18	20
1.1	0.3643	3665	3686	3708							2	4	6	8	11	13	15	17	19
					3729	3749	3770	3790	3810	3830	2	4	6	8	10	12	14	16	18
1.2	0.3849	3869	3888	3907	3925						2	4	6	8	10	11	13	15	17
						3944	3962	3980	3997	4015	2	4	5	7	9	11	13	14	16
1.3	0.4032	4049	4066	4082	4099	4115	4131	4147	4162	4177	2	3	5	6	8	10	11	13	14
1.4	0.4192	4207	4222	4236	4251	4265	4279	4292	4306	4319	1	3	4	6	7	8	10	11	13
1.5	0.4332	4345	4357	4370	4382	4394	4406	4418	4429	4441	1	2	4	5	6	7	8	10	11
1.6	0.4452	4463	4474	4484	4495	4505	4515	4525	4535	4545	1	2	3	4	5	6	7	8	9
1.7	0.4554	4564	4573	4582	4591	4599	4608	4616	4625	4633	1	2	3	3	4	5	6	7	8
1.8	0.4641	4649	4656	4664	4671	4678	4686	4693	4699	4706	1	1	2	3	4	4	5	6	6
1.9	0.4713	4719	4726	4732	4738	4744	4750	4756	4761	4767	1	1	2	2	3	4	4	5	5
2.0	0.4772	4778	4783	4788	4793	4798	4803	4808	4812	4817	0	1	1	2	2	3	3	4	4
2.1	0.4821	4826	4830	4834	4838	4842	4846	4850	4854	4957	0	1	1	2	2	2	3	3	4
2.2	0.4861	4864	4868	4871	4875	4878	4881	4884	4887	4890	0	1	1	1	2	2	2	3	3
2.3	0.4893	4896	4898	4901	4904	4906	4909	4911	4913	4916	0	0	1	1	1	2	2	2	2
2.4	0.4918	4920	4922	4925	4927	4929	4931	4932	4934	4936	0	0	1	1	1	1	1	2	2
2.5	0.4938	4940	4941	4943	4945	4946	4948	4949	4951	4952									
2.6	0.4953	4955	4956	4957	4959	4960	4961	4962	4963	4964									
2.7	0.4965	4966	4967	4968	4969	4970	4971	4972	4973	4974									
2.8	0.4974	4975	4976	4977	4977	4978	4979	4979	4980	4981									
2.9	0.4981	4982	4982	4983	4984	4984	4985	4985	4986	4986									
3.0	0.4987	4990	4993	4995	4997	4998	4998	4999	4999	5000									

The table gives $P(z) = \int_0^z \phi(z)dz$

If the random variable Z is distributed as the standard normal distribution N(0,1) then:

1. $P(o < Z < z_p) = P$(Shaded Area)
2. $P(Z > Z_p) = Q = ½ - P$
3. $P(Z^* > |Z_p|) = 1 - 2P = 2Q$